A probabilistic approach to flood hazard assessment and risk management in floodplains considering levee failures

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ABSTRACT

Levees were originally built to confine rivers to a narrow straight path and to protect the surrounding territories from floods. Those territories which originally consisted mainly of rural areas, now have become productive areas. Levees can be a defence measure against flood as well as a threat when they fail. For this reason a comprehensive flood hazard assessment in floodplains can not proceed ignoring levee failures.

Failures can occur through several mechanisms; a deterministic approach limits the result robustness, because the number of variables is high compared to available data. A probabilistic approach allows for a better handling of uncertainty, providing a robust frame to build flood scenarios.

A methodology to tackle levee failures in flood hazard assessment is proposed within a semi-probabilistic framework and applied to a 53km-section of the Adige river in South Tyrol, Italy. This methodology encompasses a probabilistic levee stability analysis, a deterministic propagation of the flood and the probabilistic combination of possible scenarios. The results are a useful tool for hazard assessment and flood risk management.

KEYWORDS

Levee failure; flood hazard; flood mitigation; semi-probabilistic approach

INTRODUCTION

Floods have recently and historically occurred in European floodplains causing damaging and casualties (Foster, 2000; Barredo, 2009). Furthermore, recently, in the Po plain, in Italy, levees have failed with serious social and economical consequences (Govi and Turitto, 2000; Moasero et al 2012; Domeneghetti et al., 2013). Embankments and levees were originally built to confine rivers to a narrow straight path and to protect the surrounding territories from floods (Aschbacher et al., 2014). Those lands which originally consisted mainly of rural areas, now have become productive and industrialized areas, where cities and infrastructures have been built. From an economical and social perspective it is clear that those areas carry a

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significant wealth which must be protected. Unfortunately experience shows that levees can be a defense
measure against flood as well as a threat when they fail (Apel et al., 2009). For this reason a comprehensive flood hazard assessment in floodplains can not proceed ignoring levee failures. This involves the development of a methodology able to detect levee weaknesses and accounting for them. The physical processes to investigate are twofold: geotechnical and hydraulic.

The breach development and formation within the levee body is determined by two factors: the hydraulic forcing and the geotechnical structure of the levees themselves, in terms of soil types and characteristics and layering conditions. (Morris et al. 2007; Morris, 2009) The first can be acquired by field investigations, such as boreholes through which soil samples can be collected and analyzed. Grain size distribution and hydraulic permeability are important parameters to assess levee stability. The latter can be investigated using in-situ penetration tests. Failures can occur through several mechanisms, such as piping, overtopping, erosion, slide of a portion of the slope, etc (Nagy and Tòth, 2005; Flor et al., 2010). In modeling the breaching process a challenge is related to understand the mechanisms with which the breach originates and how this relates to the discharge rate in the floodplain. In nature the breach failure is not an instant process, however in the modeling process this can be an assumption when the focus is on its effect on the floodplain. Given the complex reasnobable nature of the process, a deterministic approach strongly limits the robustness of the results, simply because the number of variables is too high compared to to the knowledge normally available for practical purposes (Mazzorana et al., 2009). On the contrary, a probabilistic approach would allow for a better handling of uncertainty, providing a robust frame for building scenarios.

Here a methodology is proposed to tackle levee failures in flood hazard assessment within a probabilistic framework; the methodology is then applied to a 53km-long reach of the Adige river between Terlano, Bolzano and Salorno in South Tyrol. The results proved to be a useful tool not only for hazard assessment, but also for flood risk management.

METHODOLOGY

In this section we outline the theoretical framework for the proposed methodology. In general, breaching phenomena may be triggered by several physical processes which are highly unpredictable; for this reason deterministic approaches fail to describe them appropriately. This is due to several factors, among which: the difficult assessment of the geotechnical properties of the embankment and of the subjacent soil layers, the saturation condition of the embankment, the duration of the flood event and hydrograph's shape. A probabilistic approach allows for uncertainty to be accounted for. The goal of the outlined methodology is the production of a reliable semi-probabilistic flood intensity map in a flood plain protected by earthen levees, given the hydrologic forcings and the levee characteristics (geometry and geotechnical properties), to be used as a tool for planning purposes. The semi-probabilistic flood intensity is given in terms of the maximum possible values of water depth and water velocities on the flood plain, obtained taking into account the occurrence probability of the flood, the occurrence probability of the levee failures and deterministic informations, such as the digital elevation model, levee geometry etc. The methodology is based on some reasonable assumptions: 1) breaches are statistically independent of each other; this implies that the occurrence of a breach will not influence the occurrence probability of breaches in other places along the river. This hypothesis accounts for the difficulty predicting a priori where the first breach occurs, in the context of scenarios made up of more breaches. 2) The width of the breach is assumed to be known. This parameter is estimated from statistical analyses of real breaches occurred during historical flood events (Nagy, 2006) 3) The breach is assumed to be instantaneous since for long-term maps (maximum flooding surface) short-time scale breach dynamics are not relevant (Fujita, 1987). 4)The flood hydrograph is assumed to be deterministically known in its essential characteristics, i.e. shape and peak which are estimated from data for a given return period. (Autonome Provinz Bozen, 2008)

The methodology is applied to the river of interest and to its surrounding floodplain; it requires several input data, such as the hydrological forcing (i.e. 1-in-K-flood hydrograph), the digital elevation model of the flood plain, the levee geometry and geotechnical characterization.

Levees can fail as a result of structural damage to levee itself (or to its foundation) and/or to hydraulic forces. Among all failure mechanisms two are particularly likely to occur in rivers running in floodplains and confined by earthen levees: overtopping and underseepage. Failures due to overtopping can occur when the overspilling flow erodes the levee's crest;

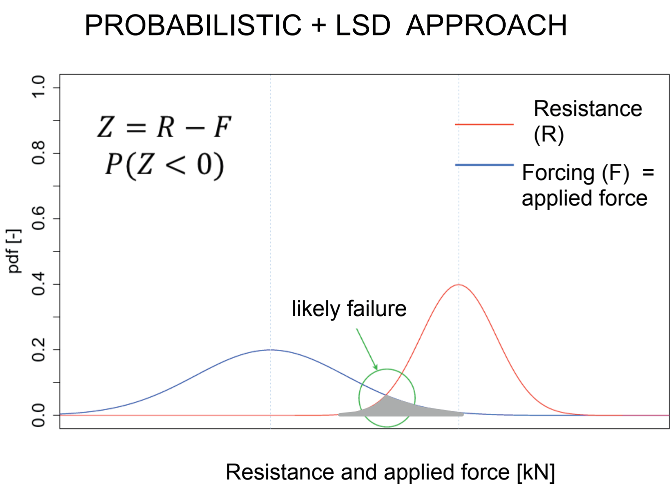


Figure 1: Probabilistic approach to compute levee failure using the LSD method.

failures due to underseepage can occur when the difference in hydraulic head between the waterside and the landside of the levee increases; as a consequence water starts to flow through the permeable foundation materials forming internal channels through erosion processes. These two mechanisms have been investigated in details and several approaches have been developed to compute the critical conditions for levees failure. In this work we chose the Vrouwenvelder (2001a) model to describe the critical overtopping discharge, the Bligh-Lane (1935) approach to identify the most critical cross sections along the river, in terms of proneness to underseepage processes, and the more detailed USACE (2000) and Sellmeijer (2006) (see Figure 3) methods to investigate those sections in details. Since geotechnical and hydraulic parameters necessary to apply the aforementioned methods are affected by uncertainty, the methodological setting was modified accordingly.

The methodology requires the computation of fragility curves for the two described mechanisms. These curves define the breaching probability for a certain cross section, given the hydraulic forcings. Fragility curves for discrete levee cross sections and for the most likely failure mechanisms are computed based on a geotechnical model built for the levee body and its foundations from available data. Fragility curve calculations are based on a resistance analysis for the earthen levee identifying a critic state beyond which the levee loses its functionality. According to the limit state design approach (LSD), the critical state is a function of the resistance force R and the applied load F. These variables (R, F) are treated as stochastic variables described by probability distribution functions (Gaussian), characterized by a mean value and by a given variance. According to this approach the critical state, i.e. the levee failure, is more likely to occur when the curves describing the probability distribution of the two variables F and R partially overlap (Figure 1). The failure probability P is thus computed as P(Z<0), where Z is the state variable Z=R-F, using

$$p(Z <) = 1 - \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\beta} e^{\frac{t^2}{2}} \mathrm{d}t; \qquad \beta = \frac{\mu_R - \mu_F}{\sqrt{\sigma_R^2 + \sigma_F^2}} = \frac{\mu_Z}{\sigma_Z}$$

equation 1

where $\boldsymbol{\mu_R}$ and $\boldsymbol{\mu_F}$ are the mean values of ***R*** and ***F*** respectively and $\boldsymbol{\sigma_R^2}$ and $\boldsymbol{\sigma_F}^2$ are the variance of ***R*** and ***F***. As to the overtopping case $\boldsymbol{Z=h_c-h_A}$, where ***Z[m]*** is the difference between the critical hydraulic head $\boldsymbol{h_c}$ ***[m]*** and $\boldsymbol{h_A}$***[m]***,the hydraulic forcing computed by the hydrodynamic model Basement (Faeh et al. 2012). hc is computed using the Vrouwenvelder (2001a) model with some simplifications due to levee characteristics.

$$q_c = \left[\frac{3.8 \cdot c_g^{\frac{2}{3}}}{(6 \cdot 10^5)^{\frac{2}{3}} \cdot \left[1 + 0.8 \log_{10} \left(P_t \cdot t_s \cdot \frac{c_g \cdot d_w}{c_g \cdot d_w + 0.4 \cdot C_{RK} \cdot L_{K,inside}} \right) \right]} \right]^{\frac{5}{2}} \cdot \frac{k^{\frac{1}{4}}}{125(\tan \alpha_i)^{\frac{3}{4}}}$$

equation 2

$$h_c = \sqrt[3]{\frac{2.78 \cdot q_c^2}{g}}$$

equation 2

The meaning of the parameters is the following: c_g ***(ms)*** is a coefficient representing the erosion endurance of grass, P_t is a percentage indicating the ratio overtopping time to flood duration, t_s ***(hours)*** is the flood duration, k is a roughness Strickler-type-factor of the riverside slope and α ***(degree)*** is the riverside slope angle. hc in equation 3 is the critical head corresponding to the critical overtopping flow q_c. The model is built on a classification of the levee's mechanical features (e.g. presence and quality of grass coating, root depth, thickness of the fine material layer) and geometry. Assigning each parameter a probability distribution function (in terms of mean and variance) instead of a deterministic value and applying the propagation error theory, a failure probability due to overtopping was calculated for each section of the levees.

For the underseepage case $Z=i_c-i_A$, $Z[-]$ is the difference between the critical gradient i_c [-] and i_A which is the hydraulic forcing gradient evaluated applying the USACE (2000) or Sellmeijer (2006) formulation. Uncertainties associated to the aforementioned hydraulic variables, i.e. h_c, h_A, i_c and i_A, originate from the simplifications adopted in the modeling approach, such as:

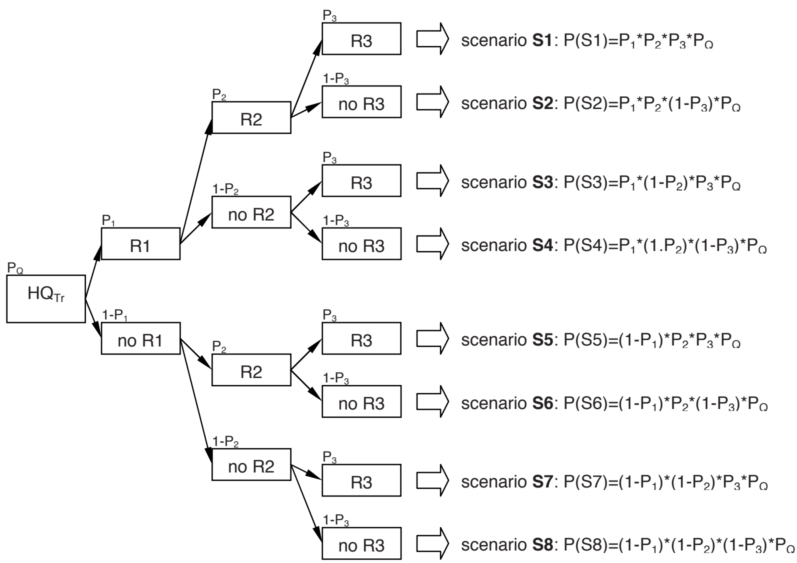


Figure 2: Sketch for the calculation of scenarios probability.

estimation of the channel roughness, approximations in the topography, estimation of the grass root depth, and spatial variability of soil parameters . Uncertainties associated to dependent variables have been computed through the propagation of independent variable errors.

For the computed N breaches, M failure scenarios are built by combining the breach formation in all possible manners (i.e. M = 2N). Given the N possible breaches, the scenario Si is characterized by the occurrence of Bi breaches and the nonoccurrence of (N-Bi) breaches. The probability of each scenario is then computed by multiplying the corresponding breach failure/non-failure probabilities, as shown in figure 2. The non-failure probability is the complement to 1 of the relative failure probability.

To compute the corresponding flow depth and the velocity components within the floodplain, for each scenario hydrodynamic simulations carried out. From a practical and operational standpoint N+1 numerical simulations have to be performed considering the N levee failures separately, the "+1" simulation refer to the case of no failure.

Since each point of the floodplain can be flooded by water coming from different breaches, the flood intensity (flow depth and velocity) is computed as the maximum water depth (and velocity) value among all simulation results which can cause a flood in that point. For example, if in the s-th scenario (out of M) a point in the floodplain (Point A) is flooded by water coming from two breaches (B1 and B2 out of N) the resulting flood map in point A is calculated as the maximum between the water depth values determined by each of the breach considered singularly (the calculation is repeated twice, first for the water depth and then for the flow velocities).

$$h_{(i,j)}^{(s)} = \max[h_{(i,j)}^{(b)}] \qquad b = 1, ...\mathrm{Y}_{\mathrm{Breaches}}^{(\mathrm{s})}$$

equation 4

where $h^{\{s\}}_{(i,j)}$ is the water depth evaluated at point (i,j) of the floodplain, s, refers to the s-th scenario, $h^{\{b\}}_{(i,j)}$ is the water depth at the point (i,j) due to the bth breach, and YsBreaches is the number of breaches, which characterizes the s-th scenario, The resulting maps displaying flow depths and water velocities, obtained by taking into account the whole set of possible scenarios, are computed as a weighted average of the local water depths (and velocity) and the associated scenario probabilities.
where H{i,j} is the water depth in the (i,j) point of the flood plain and Ps is the sth scenario probability. The same procedure is used for local velocity computations.

$$H_{(i,j)} = \frac{\sum_{s=1}^{s=M} h_{(i,j)}^{(s)} \cdot T_{(i,j)}^{(s)} \cdot P^{(s)}}{\sum_{s=1}^{s=M} T_{(i,j)}^{(s)} \cdot P^{(s)}} \qquad T_{(i,j)}^{(s)} = \begin{cases} 0 & \text{if } \; h_{(i,j)}^{(s)} = 0 \\ 1 & \text{if } \; h_{(i,j)}^{(s)} > 0 \end{cases}$$

equation 5

A CASE STUDY

The methodology was applied to a 53-km-long reach of the Adige river between Terlano, Bolzano and Salorno in South Tyrol, Italy. The Adige river was artificially altered in the XIX century and now it runs almost straight along the Adige valley. Artificial earthen levees have been built and reinforced several times during the last century; in some parts they are 7m height. In the seventies along the Adige valley a highway was built partially on embankments that sometimes run parallel to river levees. At the gauging station of Bronzolo (15 km south of Bolzano) the watershed area is 6923 km², its elevation ranges from 3905 m a.s.l to 220 m a.s.l.. The peak discharge ranges from 1400 m³/s (1-in-30-year-flood) to 1832 m³/s (1-in-200-year-flood). The flood waves typically last for 90 hours. The hydrodynamic model was calibrated for roughness using historical series of flow data.
The geometry of the Adige river had been previously surveyed and cross sections every 250 m were available. The topography of the floodplain was accurately described in order to reproduce topographical elements that can interfere with the flood, such as road and railway embankments, underpasses, etc. A good deal of data is available for the geotechnical characterization of the levees since in the last 15 years the Hydraulic Department of the Province of Bolzano (HDPB) carried out hundreds of boreholes, DPH, SPT tests and in-situ permeability tests. Furthermore laboratory analyses were carried out to obtain grain size distributions from several soil samples. The 23-km-long stretch between Terlano and Bolzano (North reach) is characterized by earthen levees with a maximum height of 3 m, whereas the 30-km-long stretch between Bolzano and Salorno (South reach) is characterized by taller levees, with a maximum elevation of nearly 7m. Observed levee failure data, collected during the last two centuries along the Adige river, suggest that in the North reach overtopping is the major cause of levee failure, on the contrary underseepage processes are predominant along the South reach.

From the available data a resistance model for both left and right levees was applied to calculate the critical overtopping flow triggering the erosion of levee crest, q_c, using equation 2 and 3 (Vrouwenvelder 2001a-b). Assigning each parameter a probability distribution function, in terms of mean and standard deviation) instead of a deterministic value and applying the propagation error theory, a failure probability due to overtopping was calculated for each section of the levees. A total of 11 levees segments were identified subjected to overtopping. Similarly, a geotechnical profile for the levees was built using data from DPH, SPT, boreholes and grain size distribution curves (Figures 4 and 5) to assign appropriate parameters to the the Bligh-Lane equation, translated into a probabilistic approach, with the aim of identifying the reaches more prone to underseepage. To those reaches the models of Sellmeijer (2006) and USACE (2000) (Figure 3 and 7) were applied, according to the sequence of subsoil layering. Hydraulic permeability values were derived from Lefranc tests.

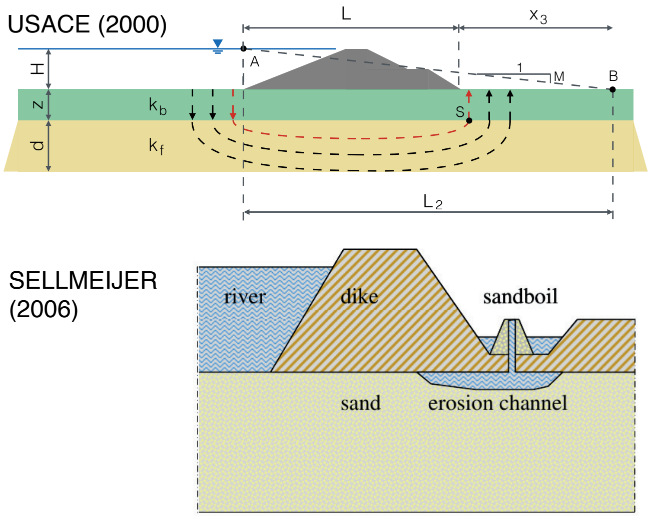


Figure 3: Sketch for the application of USACE 2000 and Sellmeijer 2006 models.

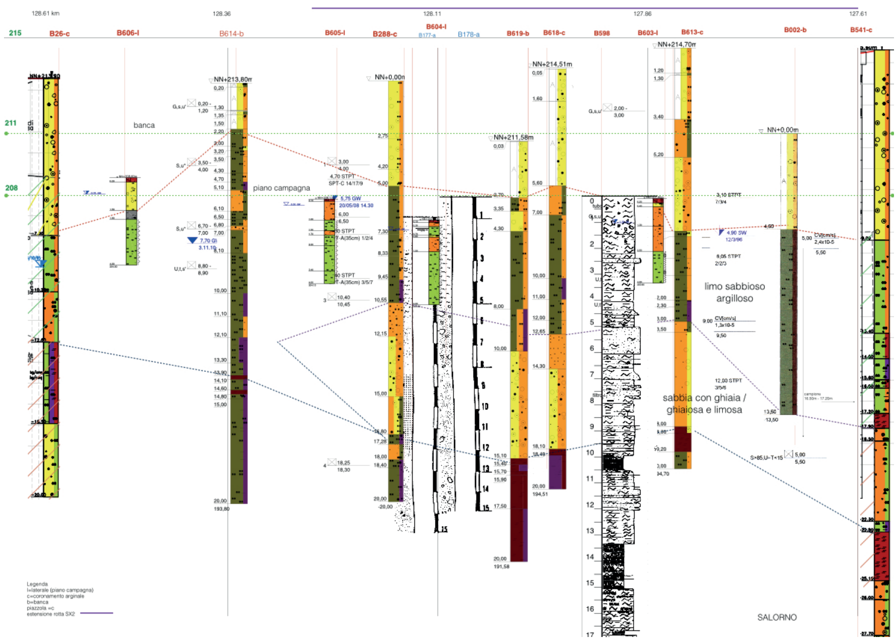


Figure 4: Geotechnical profile built on borehole and DPH data provided by HDPB for a longitudinal section along the river (failure SX2).

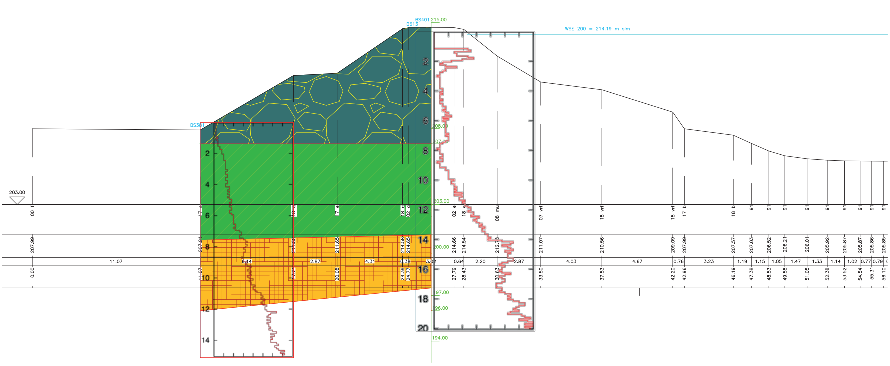
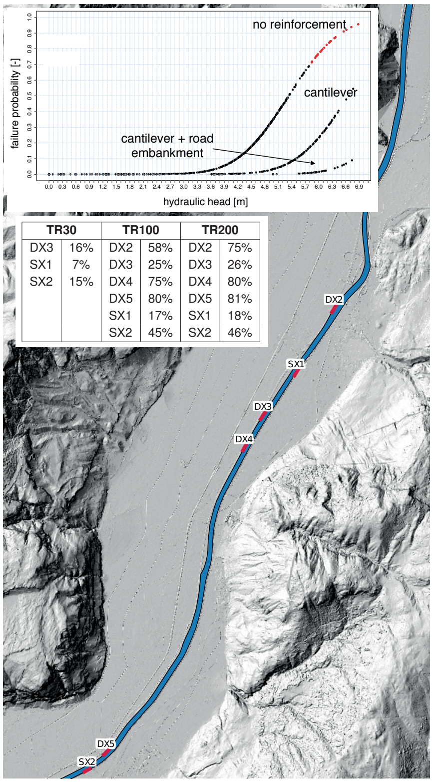


Figure 5: Geotechnical profile built on borehole and DPH data provided by HDPB for a cross section within the transect of depicted in figure 4 (failure SX2).



TR30		TR100		TR200	
DX3	16%	DX2	58%	DX2	75%
SX1	7%	DX3	25%	DX3	26%
SX2	15%	DX4	75%	DX4	80%
		DX5	80%	DX5	81%
		SX1	17%	SX1	18%
		SX2	45%	SX2	46%

Figure 6: Fragility curves for all the cross sections along the left reach of the studied river Adige portion. The three curves indicate groups of sections where the levees was reinforced with cantilevers, other where a road embankment was also present in addition to the cantilever and other which were no reinforced. Overview of the weakest points along the Adige river, South of Bolzano. Failure probability for return period of the flood and for each weak point along the South reach of the Adige river.

x	WSE TR30	WSE TR100	WSE TR200	Z_{pc}	L	D	p [%]	p [%]	p [%]
[km]	[m s.l.m.]	[m s.l.m.]	[m s.l.m.]	[m s.l.m.]	[m]	[m]	TR30	TR100	TR200
118.423	219.90	220.67	220.75	214.82	33.4	12.2	7	17	18
118.473	219.85	220.62	220.70	214.71	35.7	12.2	5	12	13
118.519	219.81	220.58	220.66	214.63	35.7	12.2	5	13	14
118.564	219.78	220.55	220.62	214.50	37.2	12.1	4	11	12
118.609	219.74	220.50	220.58	214.65	37.0	12.0	3	9	10
118.655	219.70	220.45	220.52	214.66	36.9	12.0	3	8	9
118.711	219.66	220.41	220.48	214.53	35.2	11.9	5	12	13
118.721	219.61	220.35	220.42	214.52	36.1	11.8	4	10	11
118.731	219.61	220.35	220.41	214.63	36.4	11.8	3	8	9
118.756	219.61	220.35	220.42	214.64	36.3	12.3	3	9	9
118.791	219.61	220.35	220.41	214.45	36.0	12.8	5	12	13
118.837	219.55	220.29	220.36	214.52	35.2	13.3	5	13	14
118.882	219.54	220.27	220.33	214.54	34.1	13.4	6	15	16

Figure 7: Summary of the parameters and the probability of failure computed for the failure SX1 for three return period (RP 30, 100, 200 year). x: position of the cross section, WSE: water surface elevation, Zpc: elevation of the floodplain, L: width of the left levee, D: thickness of the foundation beneath the levee, p: failure probability.

An example of longitudinal section of the geotechnical model is shown in Figure 4 for a reach along the left levee in southern part of the study area (named SX2, see Figure 6). An example of cross section along the same reach is shown in Figure 5. Applying the limit state design and a probabilistic approach, i.e. considering the limit state variables described by a pdf, fragility curves where computed for each cross section of the study stretch (Figure 6). In this analysis reinforcements of the levees were accounted for, such as cantilevers and lateral road embankment. Figure 7 displays a summary of the parameters and the probability of failure computed for the reach illustrated in Figure 5 for three return period (1 in 30, 100, 200 year flood). Figure 6 provides also an overview of the weakest points along the Adige river, South of Bolzano assigning each probable failure its probability (DX stands for right levee, SX stands for left levee). For 1-in-30-year flood failures due to underseepage can occur at 3 locations, for 1-in-100-year flood and 1-in-200-year flood the weak points are 6. The corresponding scenarios are 8 for 1-in-30-year flood, 64 for 1-in-100 and 1-in-200-year flood.
Flood map calculations have been performed applying the described methodology, considering the hydrologic forcing for each return period of the flood, the levee failure characteristics and probability. Each numerical simulation refers to a state characterized by the presence of only one levee failure at a time (b in equation 4); numerical results were then combined using equation 4 in order to evaluate flow depth and flow velocity in the floodplain for each scenario. Finally the 1-in-K-year depth and velocity maps in the floodplain were computed using equation 5 over the M scenarios identified for the 1-in-K-year-flood. These maps summarize information related to the combination of M failure scenarios. Deterministic information comes from flooding patterns, which have been analyzed in details applying a hydrodynamic model, while probabilistic information comes from the levee failures probability. The final maps represent a synthetic set of information related to the expected effects on the flood plain for a given return period of the flood.

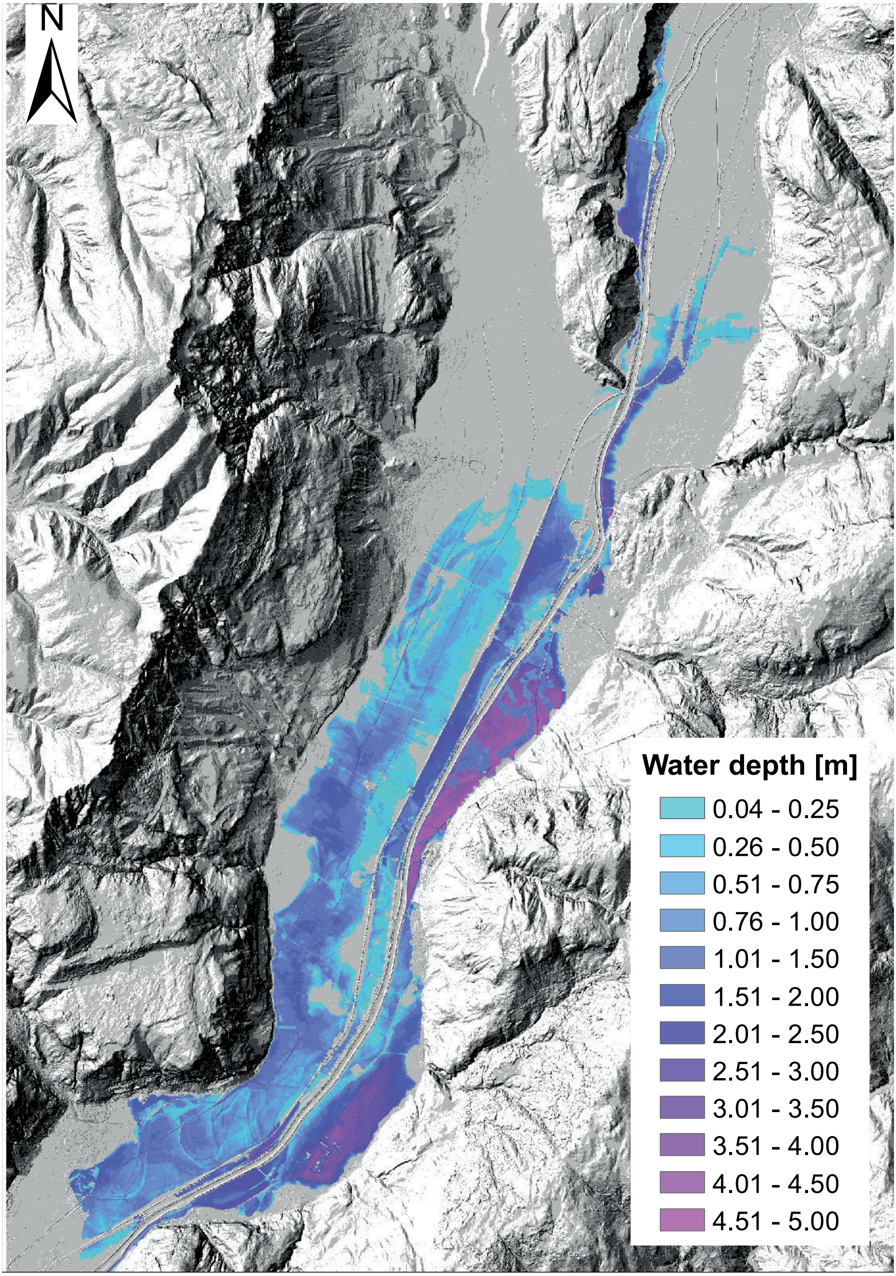


Figure 8: Semi-probabilistic water depth [m] for the Adige floodplain due to 1-in-200-year flood.

CONCLUSIONS

The evaluation of flood hazard in a urbanized floodplain is a challenge due to both the complexity of the processes involved and a shortage of detailed data often experienced in applications. Firstly, the determination of hydrological forcing relies on the availability and on the completeness of historical series of data; secondly, the evaluation of the resistance of earth embankments to high flow conditions is difficult when only few data are available to characterize geomechanical parameters; thirdly, the propagation of flooding waves due to overtopping and breach formation requires highly detailed hydraulic models. Finally, the identification of a set of scenarios able to describe a good amount of possible flood configurations is unfortunately only a simplification of the reality. The semi-probabilistic approach described in this work attempts to tackle this challenge by taking into account uncertainties hidden in the aforementioned complexity. It provides a tool to compute the flood hazard in a floodplain by synthesizing the available data and yielding maps useful for risk management and urban planning.

Results (water depth and velocity) relative to each return period of the flood, have been computed accounting for the joined occurrence of the flood and the levee failures. In contrast to a purely deterministic approach, which links effects to their own cause, these results provide a more reliable information on the actual hazard of the floodplain due to causes affected by non negligible uncertainties. The obtained flood intensity maps allow for the investigation of several levee failure scenarios in a robust manner; this provides a useful tool for flood hazard mapping and allows for appropriate risk mitigation strategies to be undertaken. It can also support decision makers and stakeholders to carry out cost benefit analysis.

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