# UNCERTAINTY IN NATURAL HAZARDS NUMERICAL MODELING

## APPLICATION OF AN HYBRID APPROACH TO DEBRIS-FLOWS SIMULATION

Guillaume Dupouy[1], **Jean-Marc Tacnet**[2], Dominique Laigle[3] and Eric Chojnacki[4]

## ABSTRACT

Natural phenomena in mountains put people and assets at risk. Risk level is often described as a combination of hazard and vulnerability. Hazard relates to the intensity and frequency of phenomena while vulnerability concerns damages and values assessment. In the debris-flows context, numerical models are used to assess height, speed and extent of flow. One important issue is to consider the influence of input data imperfection on simulation results.

This article presents the "Hybrid" approach that propagates the uncertainty through numerical simulation models and considers the different aspects of information imperfection, especially its imprecision (lack of information, inaccuracy of measure…). This new method generalizes, under some restrictive conditions, the usual Monte Carlo method, by using probability, possibility and belief function theories, used as tools for coding sets of probability densities. An example of results (quantile of deposition heights, threshold exceedance probabilities) of a numerical simulation of debris-flows is proposed: they show the influence of data imperfection including those resulting from expert assessments on the simulation results.

**Keywords**: Natural hazards, mountains rivers, debris-flows, numerical modeling, expert assessment, information quality, belief function theory, possibility theory, hybrid approach, uncertainty analysis.

## INTRODUCTION

Natural phenomena in mountains put people and assets at risk. Risk level is often described as a combination of hazard and vulnerability. Hazard relates to the intensity and frequency of phenomena while vulnerability concerns damages and values assessment. Risk assessment implies to combine those two components and propose risk reduction measures and strategies. In the debris-flows context, numerical models are used to assess height, speed and extent of flow. Others information sources such as historical data, expert assessments are also used to take a decision about the hazard level. One important issue is therefore to consider in an integrated framework the information imperfection resulting from those heterogeneous sources (figure 1).

Our problematic is to take into account more faithfully the information quality in the global hazard assessment process. The main goal of this article is to present another methodology to propagate the uncertainty through numerical simulation models based on an "hybrid" approach: we present and discuss an uncertainty analysis based on a numerical modeling of a debris-flows phenomenon.

[1] Guillaume Dupouy (M.Sc.). Cemagref , ETGR, Saint-Martin d'Hères, ENSMSE, Saint-Etienne France
[2] Jean-Marc Tacnet . PhD., Eng.Sc., Cemagref, ETGR, Saint-Martin d'Hères, France (e-mail: jean-marc.tacnet@cemagref.fr)
[3] Dominique Laigle, PhD., Eng. Sc., Cemagref, ETGR, Saint-Martin d'Hères, France.
[4] Eric Chojnacki, M. Sc., Eng., (Senior Researcher) IRSN, DPAM/SEMIC/LIMSI, Saint-Paul-lez-Durance, France.

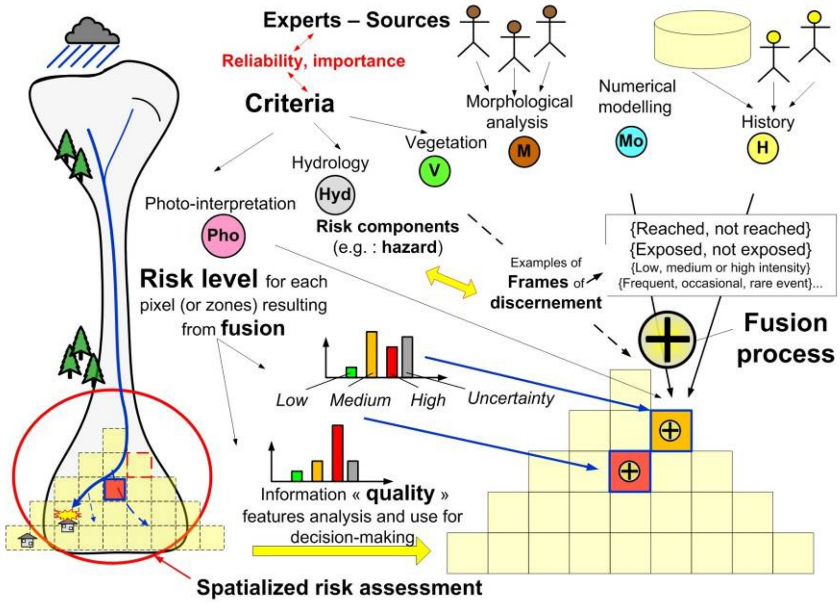



**Fig. 1** Numerical modeling is one of numerous information sources used by experts to assess risks (Tacnet et al., 2010b)

This paper is organized as follows: the first section introduces the topic. The second section briefly presents the hybrid approach and the numerical model lave2D used to simulate debris-flows. The third section describes the methodology of hybrid approach applied to lave2D calculations. Section 4 presents the application and results and section 5 is the conclusion.

## BACKGROUNDS

### Probability and possibility theories: what are the main differences?

Probability theory is the most usual and successfully completed framework to represent uncertainty. In uncertainty analysis, Monte Carlo method refers to this theory, by assessing for each input of the model, an unique density function based (ideally) on statistical samples. Unfortunately, due to partial knowledge and data, available information about input parameters remain imperfect and often come from imprecise sources (expert judgment, lack of knowledge ...). In this case, one usually chooses an unique density function among all those that don't contradict the available information. Nevertheless, that presupposes, for most imprecise variables, that one adds some information and makes a bet on the real distribution. Different choices for the same variable's uncertainty will lead to different outputs, and thus influence the decision without being clearly known by the decision maker.

### Basics of Possibility theory

Given a variable X valued in Ω, a possibility distribution is a normalized mapping $\pi : \Omega \to [0,1]$ describing the potential likelihood of the values of X, and inducing two dual measures of likelihood:

$$\Pi(A) = \sup_{x \in A} \pi(x) \quad \text{(Possibility measure)},$$

$$N(A) = 1 - \Pi(\overline{A}) \quad \text{(Necessity measure)}.$$

The possibility measure evaluates the extent to which an event A is plausible. Necessity measure evaluates the extent to which A is certain. Thus, these measures can be interpreted as bounds of the probability of A: $N(A) \le P(A) \le \Pi(A)$.

One can define an α-cut ($\alpha \in [0,1]$) as the set $E_\alpha = \{x | \pi(x) \ge \alpha\}$. Then π can be construed as a family of nested confidence sets $E_i$ with decreasing confidence degrees $N(E_i)$, corresponding to 1-$N(E_i)$ α-cuts.

$\pi$ encodes a set of probability measures (Dubois et al., 2000):

$$P_\pi = \{P | \forall A \subset \Omega, N(A) \le P(A) \le \Pi(A)\}$$

A more convenient (but less precise) representation of this set is to consider the events of type $(-\infty, x]$ and to define bounds of cumulative distribution functions (probability box).

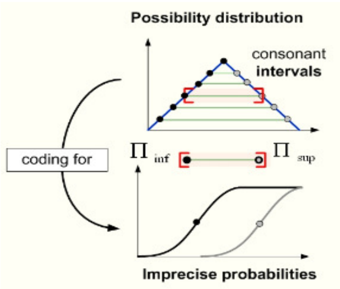



**Fig. 2** Illustration of possibility distribution and P-box.

To conclude, this theory proposes a convenient framework to represent an expert judgment as nested confidence sets with decreasing confidence degrees, and makes it possible to represent both variability and imprecision, by encoding a set of probability measures rather than one.

### Basic of Belief functions theory

One associates to a variable X valued in Ω, a mass function *m* on a finite set of subsets of Ω, that is to say, a discrete probability distribution $m : P(\Omega) \to [0,1]$. *m* induces two dual measures of likelihood (Shafer,1976):

$$Pl(A) = \sum_{E | A \cap E \neq \emptyset} m(E) \quad \text{(Plausibility measure),}$$

$$Bel(A) = 1 - Pl(\overline{A}) \quad \text{(Belief measure).}$$

The plausibility measure evaluates the extent to which the available information (traduced by *m*) implies an event A. Belief measure evaluates the extent to which the available information does not contradict A. Thus, as for possibility theory, these measures can be interpreted as bounds of the probability of A : $Bel(A) \le P(A) \le Pl(A)$. *m* encodes a set of probability measures (Dubois et al., 2000):

$P_m = \{P | \forall A \subset \Omega, Bel(A) \le P(A) \le Pl(A)\}$, that can be also represented by a probability box.

### Lave 2D: The simulation software

The lave2D model is dedicated to the computation of the unconfined free-surface spreading of materials with complex rheology (Laigle et al., 2003). It is based upon the steep-slope-shallow-water-equations which are solved using a finite volume technique which requires first to mesh the domain of interest. Equations are solved taking into account the material behaviour via a specific wall shear stress expression and the field topography represented by a Digital Elevation Model. A hydrograph can be specified as boundary condition. The model produces values of the flow height and velocity at each cell of the mesh and for each time step of the computation.

The expression of the wall shear-stress we use, due to Coussot (1994), is based upon the assumption of a visco-plastic behaviour which can be represented by a Herschel-Bulkley model. It is given by the following expression:

$$\tau_p = \tau_c \left[ 1 + 1.93 \left[ \frac{\tau_c}{K} \left( \frac{h}{\sqrt{\bar{u}^2 + \bar{v}^2}} \right)^{1/3} \right]^{-0.9} \right] \quad (3)$$

where $\tau_p$ is the wall shear stress which depends on the yield-stress $\tau_c$ and consistency K of the material. ρ is the density of the material.

A simple pre-treatment program makes it possible to declare the required inputs which are: boundary conditions (imposed hydrograph at the point where the flow enters the zone of spreading), initial conditions (flow depth and velocity at each point, if required by the case studied), computation mesh structure, elevation matrix easily imported from a G.I.S. , rheological parameters and other data relative to the computation (initial time step, CFL stability criterion of the numerical scheme, simulation duration).

The computation gives for each time and cell of the mesh, the height of material and the two components of the plane velocity. The final result is a series of matrices having the dimensions of the computation grid. Each matrix contains, for each cell: the flow height at a given time step (every 100 s for example). Each of these matrices can be directly imported into any G.I.S as a grid. In practice, the computation results are therefore used mainly to produce maps of the deposit extent, height and maximum flow depth. Such maps can be superposed on other geographical information layers, so that endangered zones can easily be delineated and compared to field vulnerability (figures 11,12,13).

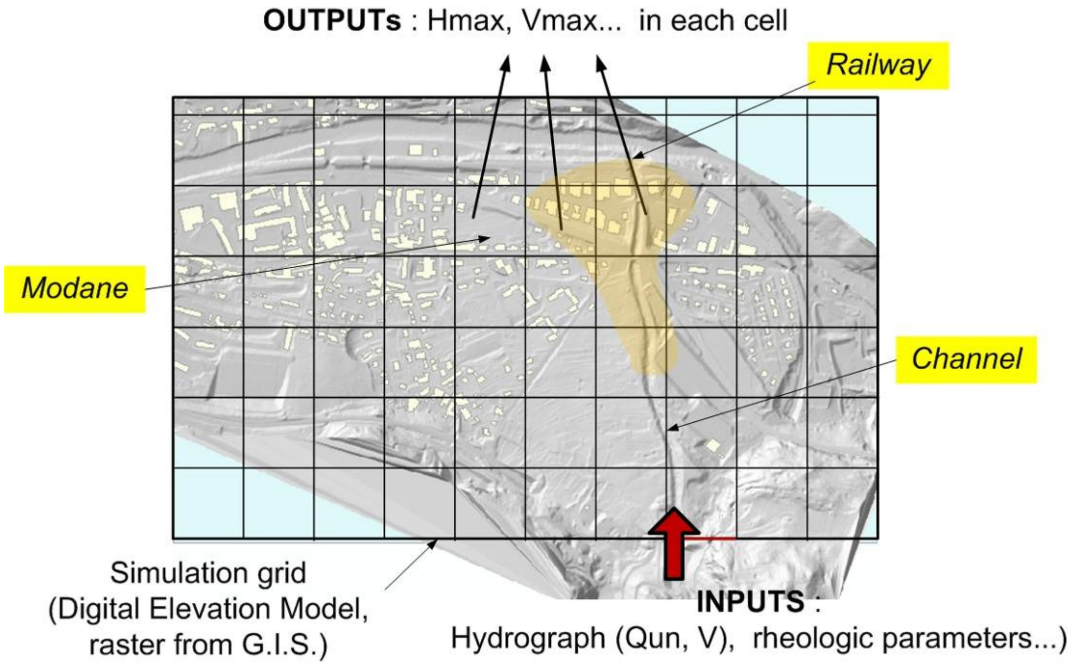



**Fig. 3** Simulation grid used by lave2D software to model the debris-flows propagation.

## METHODOLOGY

Let us consider a model $G$ with n real-valued inputs $X_1,....,X_n$ and a real valued output $Y$. The analysis of criteria of interest (quantiles, cumulative distribution function, etc.) leads to the decision.

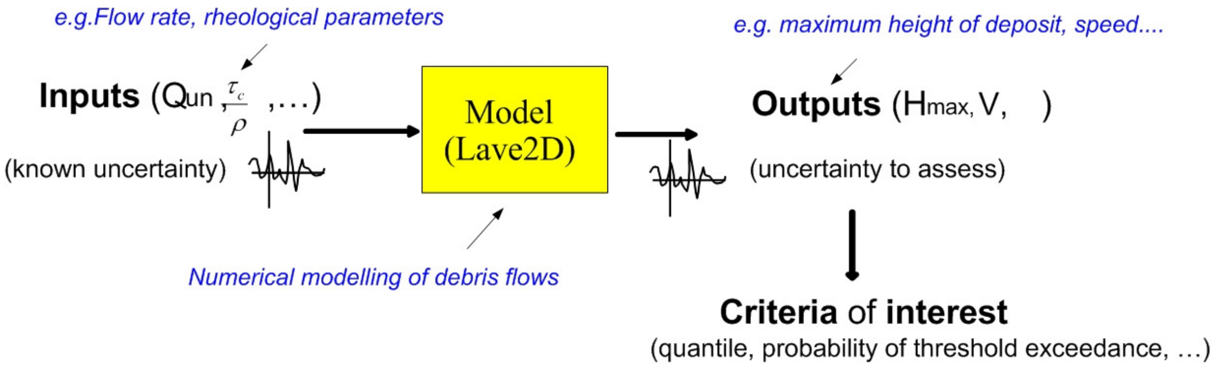



**Fig. 4** Main components of the uncertainty analysis: model, inputs, outputs and criteria of interest for results interpretation

One propagates the known uncertainty concerning the input variables through the model, assessing the uncertainty of the output variables (figure 4). Thus, the main issue is to assess at best, and as objectively as possible, the input uncertainty. In most usual uncertainty analysis approaches, probability Monte Carlo method is used through density function based on statistical samples. Unfortunately, due to partial knowledge and data, available information about input parameters remains imperfect and often comes from imprecise sources. This can correspond to assertions resulting from expert judgment such: "we are certain that this debris-flows torrential flood has reached this point... ", " it is possible that the volume will be between 20000 and 25000 $m^3$... ". In that case, the choice of a probability density function presupposes, in case of imprecise variables, that one adds some information and makes a bet on the real distribution. To consider the different aspects of information imperfection, especially its imprecision, the "Hybrid" method (Baudrit, 2005a, 2005b) (Chojnacki et al., 2009) of uncertainty analysis has been proposed: this methodology generalizes, under some restrictive conditions, the usual Monte Carlo method, by using the theories previously described: probability theory, possibility theory and belief function theory, used as practical tools for coding some imprecise probabilities (Dubois et al., 2000).

**From uncertainty of input data to uncertainty on results**

Let us suppose that information about $X_1,....,X_k$ is precise enough to be reasonably represented by its unique joint probability distribution $p_{\{1:k\}}$. $X_{k+1},....,X_n$ are imprecisely known and are respectively associated to possibility distributions $\pi_{k+1},....,\pi_n$.

- **Sampling**: one generates M samples $x^l_{\{1:k\}},....x^m_{\{1:k\}}$ stemming from $p_{\{1:k\}}$ , according to usual Monte Carlo sampling. One generates M degrees of confidence $\alpha^l,......\alpha^m$ stemming from an uniform law on [0,1], and consider the associated α-cuts $E^{\alpha^l}_{\{k+1:n\}},....,E^{\alpha^m}_{\{k+1:n\}}$ from the possibilistic variables.

1. **Optimization** and **propagation**: for each simulation $\left(x^i_{\{1:k\}}, E^{\alpha^i}_{\{k+1:n\}}\right)$ with $1 \le i \le m$, one optimizes $G$ on $E^{\alpha^i}_{k+1} \times .... \times E^{\alpha^i}_n$ to get an interval $\left[y^i_{\inf}, y^i_{\sup}\right]$ and assigns to it a mass $\dfrac{1}{M}$ : it results a mass function on Y which can be interpreted in terms of imprecise probabilities according to belief function theory (figure 5).

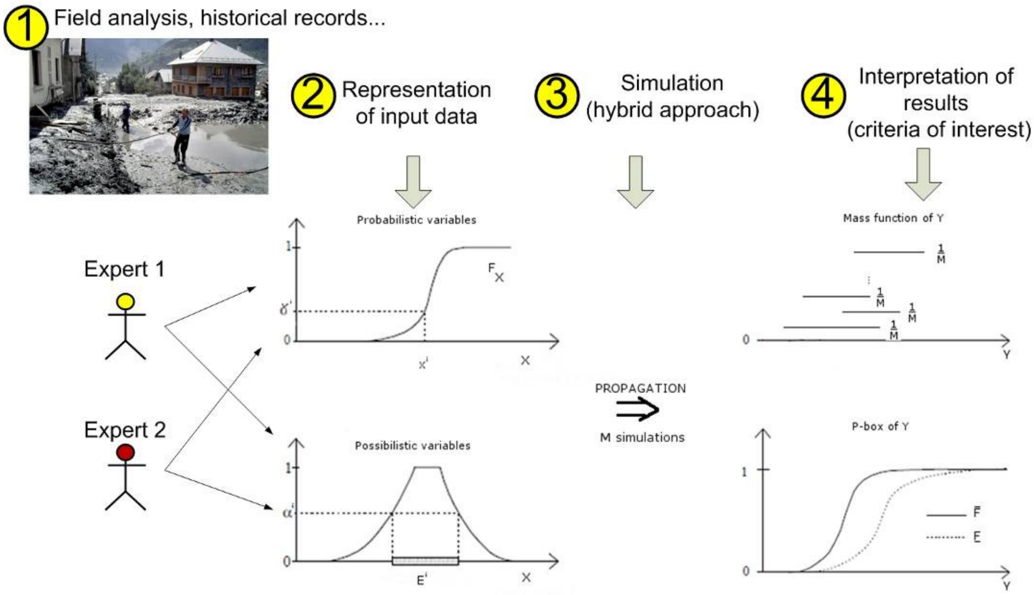



**Fig. 5** Sampling and propagation related to the Hybrid method.

5. **Post-treatment**: the numerical results induce a mass function synthesized by a P-box (lower and higher probability distributions). It can be used in an information fusion process fused considering many other imprecise sources to take a decision such as defining homogenous risk level areas (Tacnet, 2009) (Tacnet et al., 2010a, 2010b). A more global methodology is proposed to apply these methods to any simulation model and few criteria of interest, usually reached in safety studies (probability of exceedance, percentiles…) (figure 6).

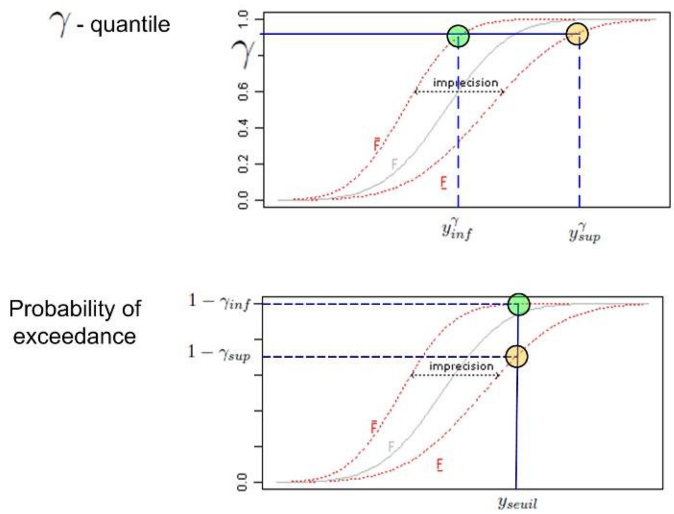



**Fig. 6** Main criteria of interest used to interpret the results: quantile and probability of exceedance.

## Restrictive conditions

In addition to the imprecision of available information about the inputs of the model, the Hybrid method requires two restrictive conditions:

1. Stochastic independence between probabilistic and possibilistic variables (as dependency structures are still unclear between a sample and an interval);

2. $G$ is monotonous for each possibilistic variable to avoid rough optimization calculi.

If any of these conditions are false for a given input, one shall resign to use an usual unique probability density function.

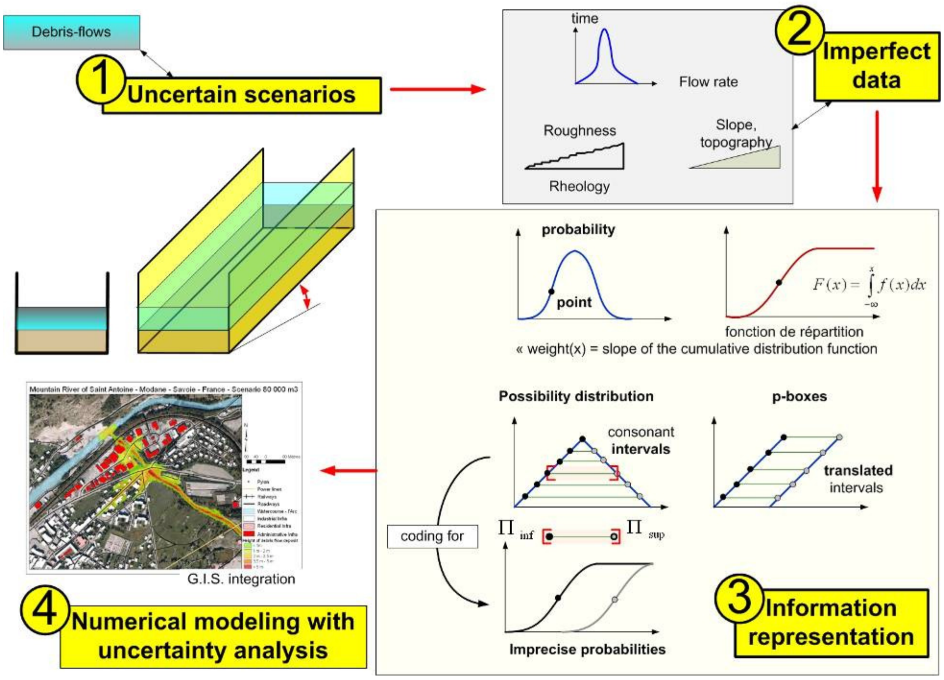



**Fig. 7** Principles of uncertainty analysis : different theories can be used to represent information imperfection.

## APPLICATIONS

### Description of the Torrent Saint-Antoine

The St Antoine Torrent is a small steep land mountain stream located near Modane in the Maurienne Valley that drains a 5.2 km² catchment. It is a left side tributary stream of the Arc river. It is exposed to three main types of floods corresponding to storm floods during warm season, snowmelt floods and floods due to the "Lombarde" regime coming from Mediterranean Sea. The catchment basin orientation is North/North-West – South/South-East and its maximum elevation is 3000 m, its minimum elevation is 1000 m. The last significant flood occurred in 1987 (55-80 000 m3 deposit volume on national road n°6. The Saint-Antoine Torrent threatens both the national road n°6 leading to the Mont-Cenis path and the railway Frejus tunnel between France and Italy (figure 8).

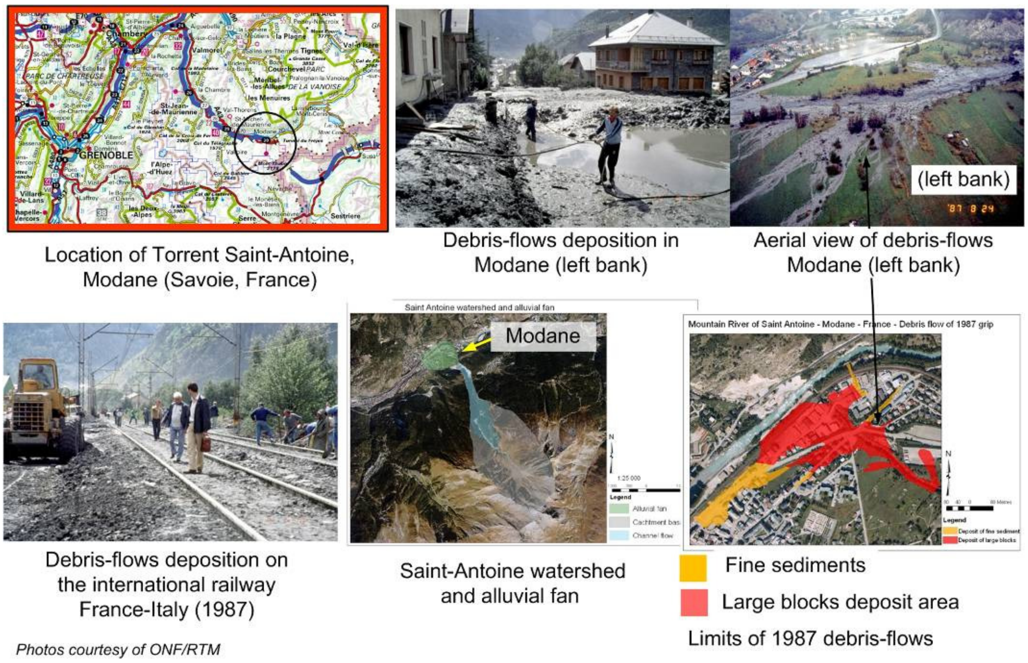



**Fig. 8** Case study: Torrent Saint-Antoine, Modane, France

**Specifications**

1. Inputs of the model

- *probabilistic variables :* rheological parameters $\frac{\tau_C}{\rho}$ and $\frac{K}{\tau_C}$ are both imprecise, but do not satisfy the condition of monotony. Stochastic independence is assumed between these variables.
- *possibilistic variables :* flow rate $Q$, time of entrance $T$ are both imprecise and satisfy the conditions of the Hybrid method.

2. Output and criteria of interest

We assessed the maximum height $H_{\max}$ in each pixel, and more especially the 0.95-quantile and the probability of the event $H_{\max} \geq 2m$.

**Quantification**

From expert assessment on the volume of debris-flow on this torrent :

"$0 \leq V \leq 170000m^3$" is certain;

"$5000 \leq V \leq 40000m^3$" is $50\%$ sure;

"$V = 10000m^3$" is the most likely.

We calculated possibility distribution for both flow rate and time of entrance (figure 9).

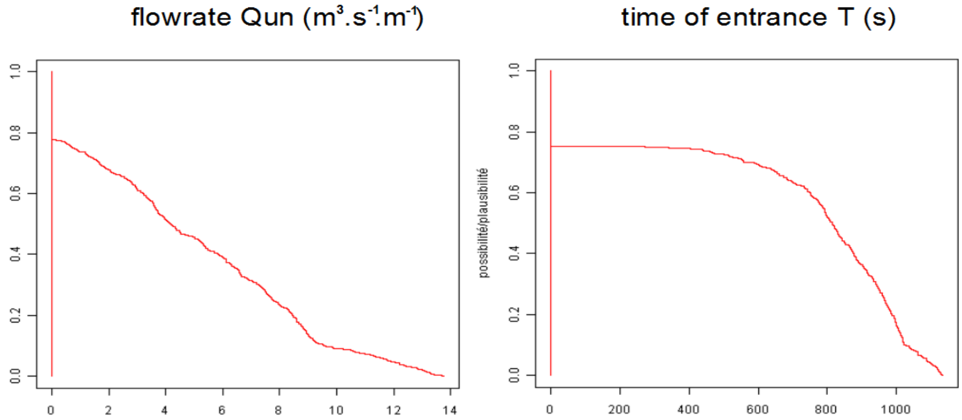



**Fig. 9** Possibility distributions for flow rate and time of entrance.

From expert assessment on the rheological parameters:

$"0.25 \leq \frac{\tau_c}{\rho} \leq 4"$ is certain ; $"0.5 \leq \frac{\tau_c}{\rho} \leq 2"$ is 75% sure; $"\frac{\tau_c}{\rho} = 1"$ is the most likely.

$"0.1 \leq \frac{K}{\tau_c} \leq 1"$ is certain ; $"0.1 \leq \frac{K}{\tau_c} \leq 0.5"$ is 75% sure; $"\frac{K}{\tau_c} = 0.3"$ is the most likely.

We chose a consistent probability density function distribution for each parameters (figure 10).

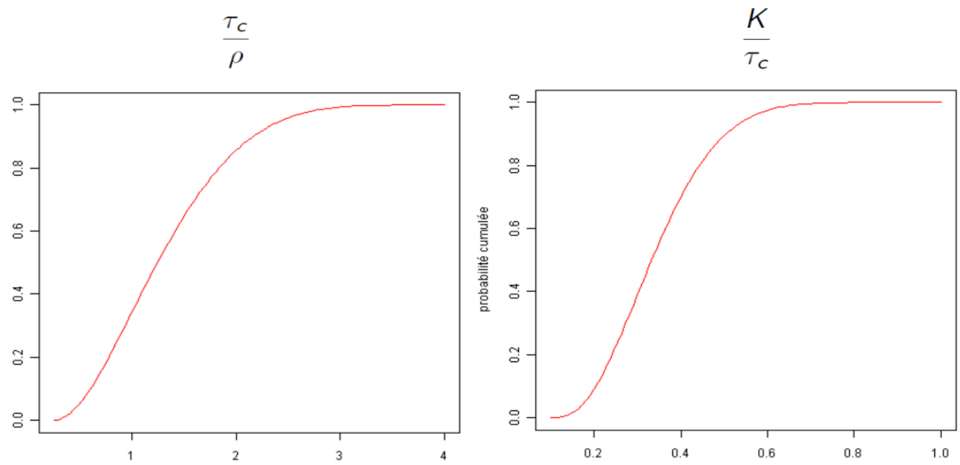



**Fig. 10** Probability cumulative distribution functions for rheological parameters.

## RESULTS

Propagation provides in each pixel a mass function on $H_{max}$, represented (for convenience) by its associated P-box, and post-treated by bounding 0.95-quantiles and probability of exceedance (figures 11, 12). The hazard assessment, so achieved, has been associated to a vulnerability analysis to lead a complete spatial risk assessment (Kaiber da Silva, 2011) (Dupouy et al., 2011) (figures 12,13).

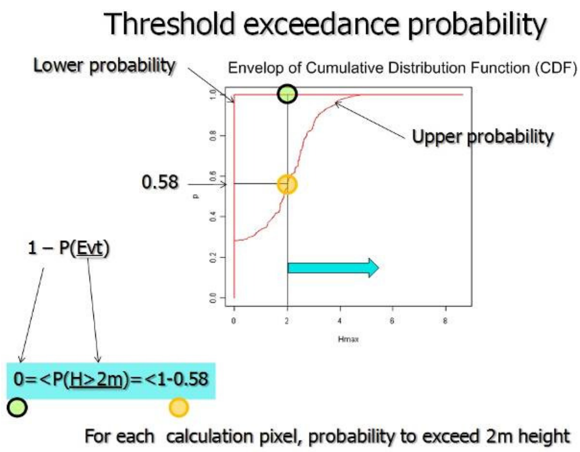
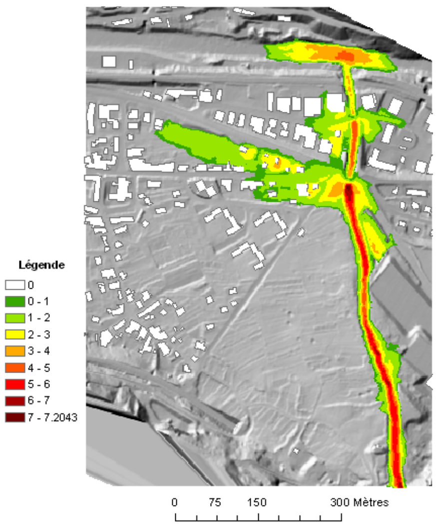
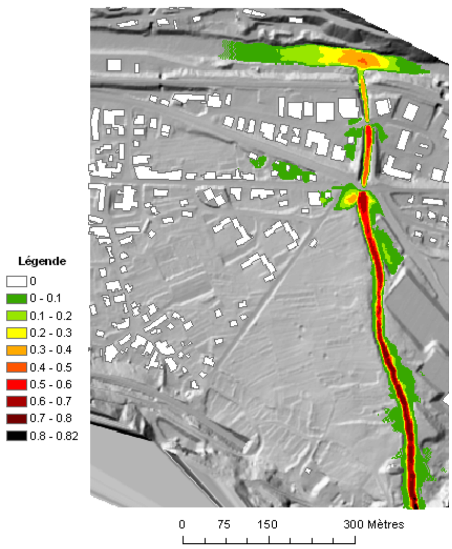



**Fig. 11** The results of numerical simulations consist in two cumulative distribution functions bounding the real but unknown because of imperfect knowledge about input data.

Numerical modeling has been done on this torrent considering different scenarios for debris flows volumes. The results are interpreted in terms of quantile 95% for the maximum height and probability exceedance (figure 11 (a) and (b)).




(a) Maximum height (in meters) reached in 95 % of cases (quantile 95)




(b) Threshold exceedance probability (a number in [0,1]) of the proposition "*Height of debris-flows deposition >2m*"

**Fig. 12** Hybrid approach: debris-flows modeling based on imprecise knowledge of input parameters (Dupouy et al, 2011)

The results are then represented in a G.I.S. and compared to different kinds of vulnerability (residential, industrial, administrative buildings) (figures 12).

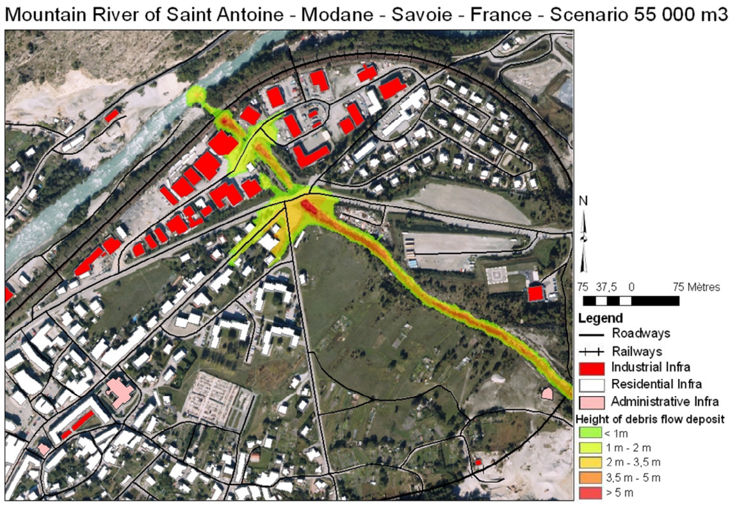



**Fig. 13** Application of hybrid approach for information imperfection assessment to the context of natural hazards (debris-flows): case study of the Paramount project – GIS representation of Scenario 55 000 m3 – quantile 95% (Dupouy et al., 2011) (Kaiber da Silva, 2011)

## CONCLUSIONS

Numerical modeling of natural phenomena is more and more used in the risk management process. Scientists, technicians and all the stakeholders involved in decision making must keep aware that any simulation model result closely depends on the quality of input data that are used. When knowledge about inputs parameters is poor, they should consider very carefully the decisions that there are going to take when they mainly found them on the numerical results. The hybrid approach is a new method to consider the real imperfection of information resulting from expert assessment.

A first application to debris-flow modeling is proposed. Possibility distributions appears to be a flexible tool for eliciting expert knowledge related to debris-flows volume, rheological parameters...The results are helpful for a spatial hazard assessment, by quantifying the heights and extension hazard through a numerical model for muddy debris flow simulation. Instead of few results, we get a wide range of simulations showing the influence of input data imperfection on results.
A key issue, which is quite new in comparison with previous applications described in the literature, is the preliminary analysis of monotonicity of the model for each parameter. Therefore, some methodological developments are still needed and under progress to adapt the hybrid approach methodology to the contexts of debris-flows and bedload transport. This also includes the key issue of results validation. Nevertheless, the results provided by this method are quite interesting for decision-makers since it shows how the real knowledge influences the simulation results. To our opinion, no numerical modeling should be done without having done this uncertainty analysis, especially when severe consequences are expected such as in the natural hazards context.

From another point of view, though the results are more realistic, it is not obvious that it really helps decision since it shows all the lack of knowledge about hazard assessment. To our opinion, it remains a great challenge to make all the stake holders aware of this. Specific developments are still required to analyze the perception of such methods and to help to take decisions on the basis of the results (Tacnet et al., 2010b).

## ACKNOWLEDGEMENTS

This research has been partially funded by the the PARAmount Project (www.paramount-project.eu) of the European InterReg Alpine Space program, the ADAGE Project (ECCOREV).


## REFERENCES


Baudrit C. (2005a). Représentation et propagation de connaissances imprécises et incertaines : application à l'évaluation des risques liés aux sites et aux sols pollués - Thèse de doctorat (PhD thesis), Université Toulouse III -U.F.R. Mathématiques Informatique Gestion, Toulouse, France.

Baudrit C., Guyonnet D. and Dubois D. (2005b). Postprocessing the hybrid method for addressing uncertainty in risk assessments, Journal of Environmental Engineering,vol.131, no.12, pp. 1750--1754.

Chojnacki E., Baccou J. and Destercke S. (2009). Numerical accuracy and efficiency in the propagation of epistemic and aleatory uncertainty. International Journal of General Systems, 0 :1-23, 2009.

Coussot P. (1994). Steady, laminar flow of concentrated mud suspensions in open channel. Journal of Hydraulic Research 32 (4): 535-559.

Dubois D., NGuyen H. and Prade H. (2000). Fundamental of Fuzzy Sets, chapter Possibility theory, probability and fuzzy sets : Misunderstandings, bridges and gaps, pages 343-438. The Handbook of Fuzzy Sets Series. Kluwer Academic Publishers, Boston, Massachussets, USA.

Dupouy G. (2010). Risques naturels et simulation numérique. Etude d'incertitude et de sensibilité par des approches hybrides : application au cas des crues torrentielles. Master's thesis, Université Paul Verlaine, Metz, France.

Dupouy G., Tacnet J.-M., Laigle D., Chojnacki E. (2011). Uncertainty analysis: application to debris flow hazard quantification in Proceedings of ROADEF 2011, 12ème congrès annuel de la Société française de Recherche Opérationnelle et d'Aide à la Décision, 2-4 mars, Saint-Etienne, France, 2011.

Kaiber da Silva J. (2011). Integrated scenarios analysis and decision support for managing risks associated with torrential floods – Projet de fin d'études mention Management des Risques et Environnement – Ecole des Mines d'Alès, Cemagref (Grenoble), France.

Laigle D., Hector A.-F., Hübl J., Rickenmann D. (2003). Comparison of numerical simulation of muddy debris flow spreading to records of real events, Proceedings of the 3rd International Conference "Debris-Flow Hazards Mitigation", Davos, Switzerland, 10-12 september 2003, Edited by D. Rickenmann & C.-l. Chen, MILLPRESS 2003, 635-646.

Shafer G. (1976). A mathematical theory of Evidence, Princeton University Press.

Tacnet J.-M. (2009). Prise en compte de l'incertitude dans l'expertise des risques naturels en montagne par analyse multicritères et fusion d'information. Thèse de doctorat (PhD thesis), Ecole Nationale Supérieure des Mines de Saint-Etienne, 2009.

Tacnet J.-M., November V., Richard D., Batton-Hubert M., (2010a). Expert assessmen, decision and uncertainty : how far is an interdisciplinary approach possible in the context of integrated management of natural risks in mountains? (in French) , International Conference OPDE 2010, 25-26 october 2010, Montpellier, France.

Tacnet J.-M., Rosalie M., Dezert J., Travaglini E. (2010b). Spatial information fusion: application to expertise and management of natural risks in mountains, SAGEO'10 Conference, Toulouse, France, 17-19 November 2010.