TWO DIMENSIONAL MODELLING OF SNOW TEMPERATURE AND SNOW SETTLEMENT

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ABSTRACT

The investigation of properties of the snow pack aims to a very important field of snow science. The knowledge of snow temperatures provides the basis of any further calculations of physical processes within the snow pack. Therefore a new two dimensional snow pack model is presented which calculates snow temperature deviations and snow densities along a two dimensional cross section of the snow pack. The evaluation of the simulated snow temperatures presents different temperature gradients, depending on snow depth, slope inclination and exposition. The model allows the import of digital elevation models generated by terrestrial or airborne laser scans. All necessary evaluation- and input data are measured by automatic weather stations.

Keywords: snow pack modelling, Finite Element Method, heat transfer, snow settlement

INTRODUCTION

One part of snow science deals with the investigation of snow pack properties. There exist a number of physical snow pack models like *Snowpack* which was developed by the SLF Davos or *Safran* and *Crocus* developed by Météo-France. These models are able to predict the evolution of the snow pack and its stability. Unlike to the existing models, a two dimensional snow pack model was developed at the Institute of Mountain Risk Engineering. This model allows the calculation of snow temperatures, settlement and densification of arbitrary chosen cross sections of a slope. The model describing differential equations are solved with the Finite Element Method which is implemented in Matlab.

GENERATION OF TWO DIMENSIONAL CROSS SECTION GEOMETRIES

Digital elevation models generated by air born and terrestrial laser scans provide a basis for the geometry definition of a two dimensional snow pack model. The pre processing mask of the simulation program allows the definition of any arbitrary intersection of a slope. To draw a two dimensional cross section geometry, snow depth along this intersection can be measured punctually by a supersonic measuring device or laminary by dint of terrestrial laser scans. Figure 1 shows an example for choosing a two dimensional cross section and the extracted cross section.

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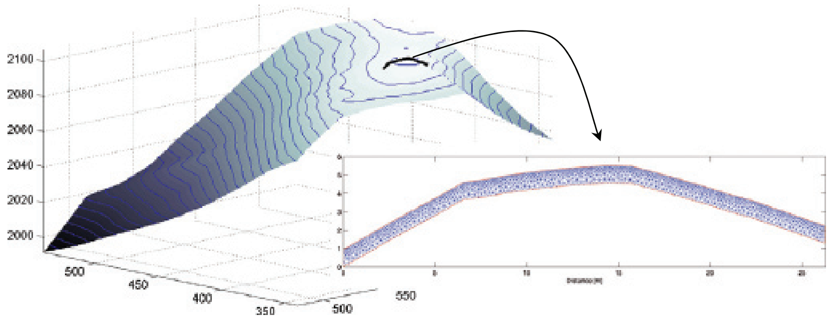

Fig. 1: Generation of a snow pack's cross section geometry. The slice along the black line is triangulated as shown in the right figure.

A delaunay-algorithm is used to triangulate the cross section geometry. The mesh size near the snow surface is smaller than near the soil in order to get better calculation results of thermal fluctuations near the surface, influenced by atmospheric conditions and penetrating solar radiation. A discretisation of the geometry is required for the Finite Element Method which solves the partial differential equations of heat transfer, mass balance and snow settlement numerically.

EXTRAPOLATION OF MEASURED DATA

A two dimensional snow pack model requires the extrapolation of automatic gauging station measurements to other points of the surrounding area. Some data, for example air temperature or humidity will vary just a little within a narrow radius of the gauging station. Compared to this, there are long wave emission-data which are highly dependent on the snow-surface-temperature and the portion of short wave irradiation which depends on exposition and inclination of the slope.
For a precise snow pack simulation exact input data are required. However, some measuring instruments react very sensitive to atmospheric influences and often provide unusable measurements at unfavourable weather conditions. For this reason the test series must be checked and mistakes must be corrected before a simulation can be started. Therefore Matlab smooth functions can be used, to filter erroneous measurements.

Short wave radiation

The calculation of the wave angle φ between solar irradiation and the terrain's normal vector is essential for the calculation of the energy input. By means of the wave angle the percentage of the measured short wave radiation which hits the particular area can be calculated on every single day of the year at any time. Furthermore it can be derived if a calculation point is located in the shadow of another point by using a digital elevation model. The intensity $I(\varphi)$ of the incoming solar radiation is defined as $I(\varphi) = \cos(\varphi)$.

Short wave radiation is measured by using two pyranometers. One pyranometer aiming downwards measures the reflexion of short wave radiation on top of the snow surface whereas the second pyranometer aiming upwards measures the incoming radiation to a horizontal area. The albedo a refers to the quotient of irradiation and emittance. The measurement of shortwave radiation can be distorted by alpine weather conditions like snow fall or hoarfrost. Unfortunately there is no possibility to calculate short wave radiation instead of measuring it.

The only way to check the measured data is to compare incoming and reflected radiation. If unrealistic albedo values arise, an error of measuring should be considered. Since the measurements of the lower part of the radiation measuring device are often more reliable a relationship to the radiation can be established by means of the albedo.
The punctual measurement of shortwave radiation has to be extrapolated from the spatial point of the weather station to the surrounding alpine topography. Therefore the angle φ_{ref} defines the angle between the normal vector of a horizontal plain and the incoming solar radiation. The amount of energy E_{in} which finally arrive the snow surface is calculated as follows:

$$kw_{ref} = \frac{kw_{in}}{\cos(\varphi_{ref})}$$

$$E_{in} = (1-a) \cdot I(\varphi) \cdot kw_{ref} \tag{1}$$

The short wave reference value kw_{ref} represents the amount of radiation measured by a pyranometer which would be permanently directed normal to the incoming sun rays. It can be calculated including the pyranometer measurement of the incoming radiation kw_{in} and the current position of the sun, which is derived for each time step of the simulation. The calculation of the reference radiation includes a division through $cos(\varphi_{ref})$ which is nearly zero at sunset and sunrise and exactly zero during the night. To avoid complications at the division through extremely small numerators and denominators the calculation of kw_{ref} is replaced by a bell-shaped curve during sunrise and sunset.
Figure 2 shows the different intensities of incoming shortwave radiation, dependent on inclination and exposition. Besides intensities, the program calculates shadows too which are plotted dark blue.

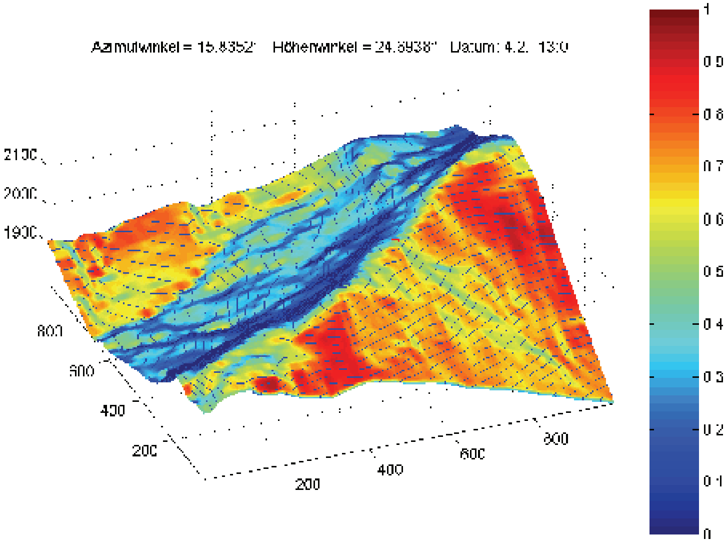

Fig. 2: Calculation of intensity I and shadows, (2005/02/04; 13:00)
red = 100% intensity; blue = 0% intensity

Long wave radiation

The measurement of long wave radiation is carried out with nonventilated pyrradiometers, which are very sensitive against atmospheric conditions. Thus it has been tried to replace the measured values by calculated ones. Thereby it is paid attention that only robust data are used for the calculation. Long wave emittance lw_{out} can be calculated as a function of the snow surface temperature T_s [°K], using the Stefan Boltzmann Equation:

$$lw_{out} = \sigma \cdot T_s^{\,4} \quad with \;\; \sigma = 5.669 \cdot 10^{-8} [\tfrac{W}{m^2}] \tag{2}$$

Calculating the long wave irradiation lw_{in} is more difficult. The long wave irradiation equation, developed by the author, strongly depends on air temperature, air humidity and sky cover. Since sky cover cannot be measured, the calculation is limited to an empiric link of air humidity rh and air temperature LT [°K].

$$lw_{in} = 0.8 \cdot \frac{\min(\max(35, rh), 85)}{100} \cdot \sigma \cdot LT^4 + 90 \tag{3}$$

The net long wave radiation lw is a result from the difference between irradiation and emittance. It is required later to calculate the energy flux over the snow surface. A comparison between measured balance and calculated balance shows that the calculation comes qualitatively close to the measurement but quantities do not fit exactly (fig. 3).

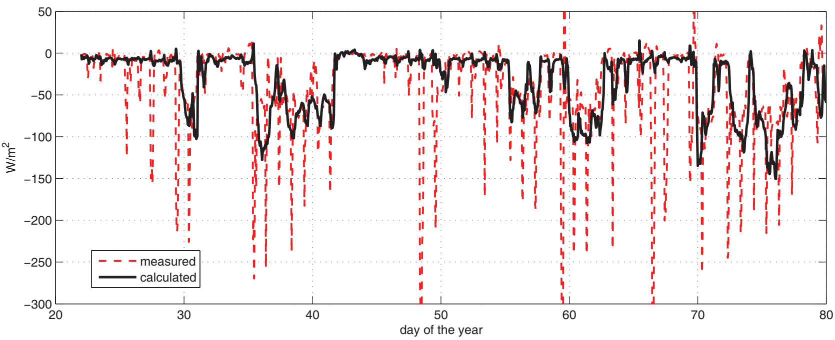

Fig. 3: Comparison between measured and calculated long wave radiation balance between 2005/01/22 and 2005/03/21

MODELLING OF HEAT TRANSFER WITHIN THE SNOW PACK

Heat transfer within the snow pack can be derived by the heat equation. This equation requires the knowledge of snow density ρ_s, specific heat capacity of snow c_s, thermal conductivity k_s and a source term Q, which describes the influence of short wave radiation penetrating the upper 30 to 40 cm of the snow pack.

$$\rho_s c_s \frac{\partial T(x,y,t)}{\partial t} = \nabla \bullet (k_s(x,y,t)\nabla T(x,y,t)) + Q(x,y,t) \tag{4}$$

The source term Q plays an important role for modelling snow temperatures near the snow surface. The source term Q is described by the law of Beer-Lambert which relates the absorption of light to the properties of the material through which the light is traveling. The following equation defines the relation between incoming short wave energy E_{in} and the penetration depth p_d. u_1 and u_2 are empirical parameters depending on the material properties.

$$Q(p_d) = u_1 \cdot E_{in} \cdot \exp(-u_2 \cdot p_d) \tag{5}$$

The boundary conditions of the heat equation are divided into three different types. If the temperature on the surface boundary of the area is known one should prefer to use Dirichlet-boundary-conditions Γ_D. A possibility to describe a heat flux between snow surface and atmosphere is offered by Neumann-boundary-conditions Γ_N or rather hybrid-boundary-conditions Γ_H. If using this hybrid condition, not only heat flux via atmospheric radiation is considered, but also convective influences just like temperature difference between environment and surface coupled with wind velocity.

$$\begin{aligned} T &= T_D && on\ \Gamma_D \\ k_s \frac{\partial T}{\partial \vec{n}} &= g && on\ \Gamma_N \\ k_s \frac{\partial T}{\partial \vec{n}} &= g + v \cdot (T_{ext} - T) && on\ \Gamma_H \end{aligned} \tag{6}$$

Different boundary conditions, depending on the available measurement data and symmetry conditions are assigned to the four separated boundaries of the cross section. The left and right boundaries are defined by symmetric conditions which imply homogeneous Neumann-boundary-conditions. The interface between the snow pack and the soil is defined by a Dirichlet-boundary-condition. Temperatures at the bottom of the snow pack are nearly constant at zero degrees Celsius all over the winter. The surface of the snow pack can be described either by a Dirichlet-boundary-condition if snow surface temperature measurements are available or by a hybrid-boundary-condition if energy fluxes are used to derive the energy balance between snow pack and atmosphere. Therefore detailed measurements and calculations of long wave radiation, wind speed, air temperature and relative humidity are essential for the parameterization of the hybrid-boundary-condition. The following equations show the constitution of the hybrid-boundary-condition and the influence of long wave radiation, latent- and sensible heat fluxes.

$$k\frac{\partial T}{\partial \vec{n}} = lw + q_{latent} + q_{sensible}$$

$$lw = lw_{out} - lw_{in}$$

$$q_{sensible} = f(v_{wind}) \cdot (T_{air} - T_s)$$

$$q_{latent} = n \cdot v_{wind} \frac{L^{i/w} \cdot \rho_{air}}{p_{air}} (e_s^w(T_{air}) \cdot rh - e_s^i(T_s)) \tag{7}$$

$$e_s^{i/w}(T) = p_t \cdot \exp(\frac{L^{i/w}(T - T_t)}{R_v \cdot T_t \cdot T})$$

$L^i = 2838$ *[kJ/kg]*...........latent heat of sublimation

$L^w = 2256$ *[kJ/kg]*..........latent heat of vaporization

rh...................................relative humidity

v_{wind}................................wind speed

$p_t = 610,5$ *[Pa]* ....triple point pressure

$T_t = 273,16$ *[K]* ....triple point temperature

$R_v = 461,9$ *[J/(kg K)]*.....specific gas constant

n.....................................empirical constant

The partial differential equation of heat transfer is solved by the Finite Element Method, based on a triangular meshed cross section. The algorithm is designed to handle time variable geometries of the cross section, caused by snow drift, settlement and melting. Further details of the settlement of the snow pack and the influence of the Finite Element mesh are described in the next section.

MODELLING SNOW SETTLEMENT AND DENSIFICATION

It is assumed that the settlement is caused by the snow pack's own weight. Therefore the creeping of the snow pack is modelled by the equilibrium condition of the plane strain. The body force (q_x, q_y) represents the force acting on each grid point, respectively each mass point in x- and y-direction. The stress tensor σ is defined as a Newtonian fluid with very high viscosities. In the following section $\dot{\varepsilon}$ defines the strain rate and η the viscosity.

$$\dot{\varepsilon} = \frac{1}{\eta}\sigma \tag{8}$$

$$div\ \sigma = -q \quad \Leftrightarrow \quad \nabla \bullet \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = -\begin{pmatrix} q_x \\ q_y \end{pmatrix}$$

$$\Leftrightarrow \quad \underbrace{\begin{pmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix}}_{=d^T} \cdot \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = -\begin{pmatrix} q_x \\ q_y \end{pmatrix}$$

$$\varepsilon = d \cdot u \quad \Leftrightarrow \quad \dot{\varepsilon} = d \cdot \dot{u}$$

$$\Leftrightarrow \quad \begin{pmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ 2\dot{\varepsilon}_{xy} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \cdot \begin{pmatrix} \dot{u}_x \\ \dot{u}_y \end{pmatrix} \tag{9}$$

The influence of snow metamorphism is not modelled explicitly, but it is implicated indirectly at the definition of the snow viscosity. The viscosity η is derived empirically, described in the following equation.

$$\eta = \begin{cases} (h_1 \cdot \rho)^{(h_2 - h_3 \cdot T)} & \text{for } \rho < \rho_g \\ \left(\dfrac{\rho_s}{\rho_g}\right)^5 \cdot (h_1 \cdot \rho)^{(h_2 - h_3 \cdot T)} & \text{for } \rho \geq \rho_g \end{cases} \tag{10}$$

ρ_g *~150 kg/m*^{3} is a threshold density, where the rate of settlement decreases very fast. The remaining parameters h_1, h_2, h_3 are describing the influence of temperature T_s (in [°C]) and snow density ρ_s.

As a consequence of the settlement snow densities arise. The densification of the snow pack can be calculated by dint of the mass conservation equation applied on compressible materials.

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho\, \dot{\vec{u}}\right) = 0$$

$$\dot{\vec{u}} \text{.... velocity of settlement} \tag{11}$$

The calculation of the snow settlement is performed by a Lagrangian-description what means, that the Finite Element mesh deforms equivalent to the real mass points. From this it follows, that the shape of the cross section and the mesh is calculated by the settlement calculation. Therefore no remeshing has to be performed to adapt the mesh to the temporally changing cross section. In contrast, the consideration of snow melt and fresh fallen snow demands remeshing and mapping operations because elements are added or removed, respectively.

EVALUATION

Simulation results are compared with measurements recorded by automatic gauging stations installed in *Lech am Arlberg*. One station is placed on a northern slope and the other one is south-east exposed. A data logger saves every 10 minutes long- and shortwave radiation, air temperature, relative humidity, wind speed, snow depth, soil temperature, snow surface temperature and temperatures within the snow pack. The continuous measurements of atmosphere and snow pack conditions allow the evaluation of simulated snow temperatures within the snow pack and on its surface. The following plot (fig. 4) shows the comparison between measured and calculated snow temperatures at the south-east exposed gauging station. The dotted lines are measured values and solid lines are simulation results. The different colours are representing measurements in different snow depth: green *32 cm,* red *52 cm* and blue *72 cm* over soil.

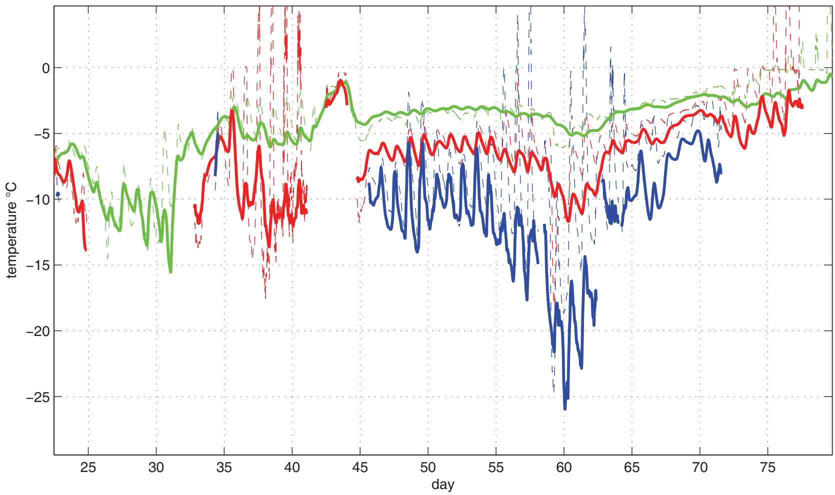

Fig. 4: Calculated and measured snow temperatures in three different heights between 2005/01/23 and 2005/03/21

Figure 5 shows the comparison between measured and calculated snow surface temperatures. The simulation was performed under consideration of the calculated long wave radiation, not measured radiation. The comparison with measured surface temperatures shows very good correlations.

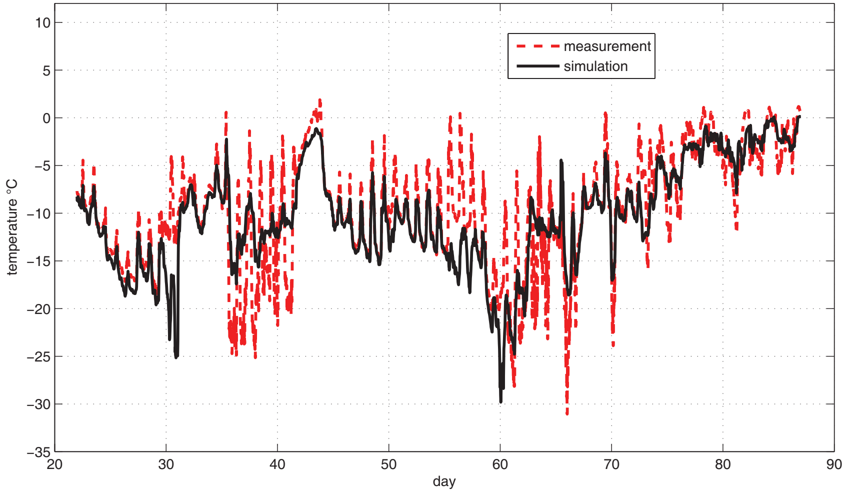

Fig. 5: Comparison of calculated and measured snow surface temperatures between 2005/01/22 and 2005/03/24

Besides the pointwise evaluation of snow temperatures, the two dimensional snow pack simulation gains insight of the temperature deviation along a slice of a snow covered slope. The following figure 6 presents temperature differences caused by different expositions, inclinations and snow depths.

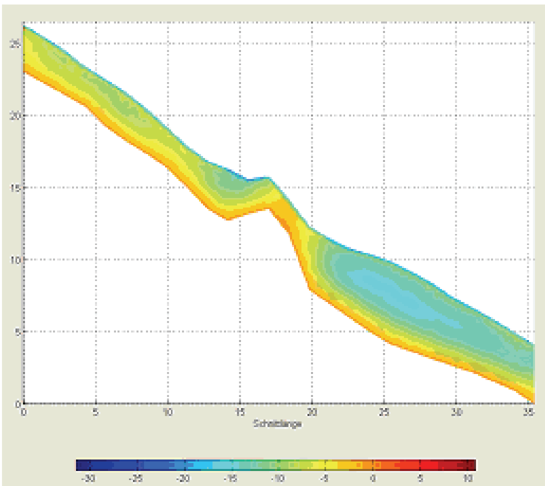

Fig. 6: Temperature deviation inside the snow pack, evaluated on the 2005/02/05 at 10:30 (red ~ 0°C, cyan ~ -15°C, dark blue ~ -30)

A comparison between simulated and measured snow depths is given in figure 7. The evaluation presents good correlations besides at the end of the winter. The real snow depth are overestimated. This effect was expected, because there's no melting model implemented till now.

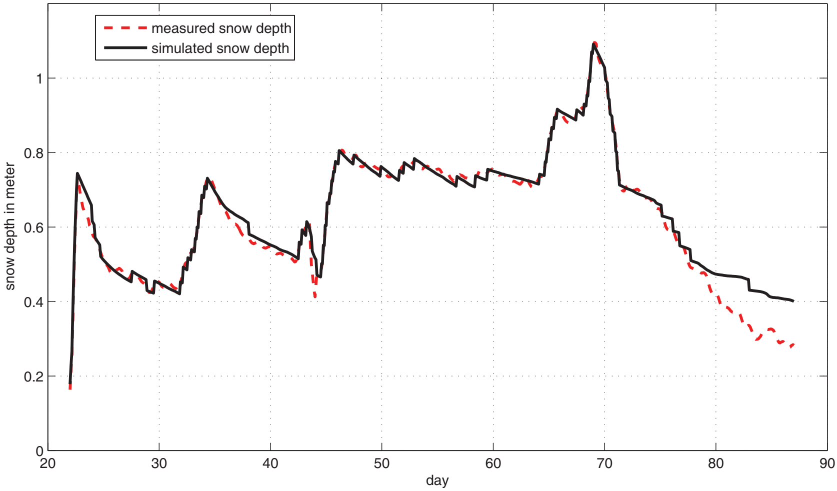

Fig. 7: Comparison between simulated and measured snow depth between 2005/01/22 and 2005/03/24

OUTLOOK

There are still a lot of enhancements that can be done, like the modelling of snow metamorphism, melting processes and surface hoar. The two-dimensional snow cover model will be linked to a wind simulation to get better input data for the simulation. Further improvements are expected by terrestrial laser scan technology, which allows a three dimensional scanning of mountainsides. By dint of this new technology more precise snow pack cross sections can be measured.

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